

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2

Semester : 1

Date : 28/11/2018

Branch: B. Tech (All)

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) If $y = \frac{1}{x}$ then y_n equal to
(A) $\frac{(-1)^n n!}{x^n}$ (B) $\frac{(-1)^n n!}{x^{n+1}}$ (C) $\frac{(-1)^{n-1} (n-1)!}{x^n}$ (D) None of these
- b) If $y = (10)^{2x}$ then y_n equal to
(A) $2^n (10)^{2x} (\log 10)^n$ (B) $2^n (10)^{2x} (\log 2)^n$ (C) $2^n (10)^{2x} (\log 10)^{n+1}$
(D) none of these
- c) The series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ represent expansion of
(A) $\cot^{-1} x$ (B) $\tan^{-1} x$ (C) $\sin^{-1} x$ (D) $\sin x$
If $y = \log(1+x)$, then x equal to
(A) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$ (B) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-\dots$ (C) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+\dots$
(D) none of these
- e) $\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = \underline{\hspace{2cm}}$
(A) 0 (B) 1 (C) 2 (D) none of these
- f) $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} = \underline{\hspace{2cm}}$
(A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log\left(\frac{5}{3}\right)$
- g) If $P = r \tan \theta$, then $\frac{\partial P}{\partial r}$ is equal to
(A) $\sec^2 \theta$ (B) $\tan \theta$ (C) $\tan \theta + r \sec^2 \theta$ (D) $\frac{1}{2} \tan \theta$



- h)** If $x = r \cos \theta$, $y = r \sin \theta$, then $\frac{\partial r}{\partial x}$ is equal to
 (A) $\sec \theta$ (B) $\sin \theta$ (C) $\cos \theta$ (D) $\cosec \theta$
- i)** If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- j)** If $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$ is equal to
 (A) 1 (B) -1 (C) zero (D) none of these
- k)** If $y = \cos \theta + i \sin \theta$, then the value of $y + \frac{1}{y}$ is
 (A) $2 \cos \theta$ (B) $2 \sin \theta$ (C) $2 \cosec \theta$ (D) $2 \tan \theta$
- l)** If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$ then $x_1 x_2 x_3 \dots \text{to } \infty$ is
 (A) -3 (B) -2 (C) -1 (D) 0
- m)** An eigenvalue of a square matrix A is $\lambda = 0$. Then
 (A) $|A| \neq 0$ (B) A is symmetric (C) A is singular (D) A is skew-symmetric
- n)** The product of the eigenvalues of $\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$ is
 (A) 2 (B) 4 (C) 6 (D) 0

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

- a)** If $y = \frac{x}{x^2 + a^2}$ then find \mathbf{y}_n . (5)
- b)** Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)
- c)** If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$. (4)

Q-3

Attempt all questions

(14)

- a)** If $y = \sin(m \sin^{-1} x)$ then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$
- b)** Prove that $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$ (5)
- c)** Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a}{x} - \cot \frac{x}{a} \right)$ (4)

Q-4

Attempt all questions

(14)

- a)** Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ (5)



b) If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, evaluate $J = \begin{pmatrix} x, y \\ u, v \end{pmatrix}$ and $J' = \begin{pmatrix} u, v \\ x, y \end{pmatrix}$ and hence verify that $JJ' = 1$. (5)

c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$. (4)

Q-5 **Attempt all questions** (14)

a) If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. (5)

b) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x)\sec 2x$ (5)

c) Find nth derivative of $\tan^{-1} x$. (4)

Q-6 **Attempt all questions** (14)

a) Using the formula $R = \frac{E}{I}$, find the maximum error and percentage of error in R if $I = 20$ with a possible error of 0.1 and $E = 120$ with a possible error of 0.05 and $R = 6$. (5)

b) Prove that $(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$. (5)

c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. (4)

Q-7 **Attempt all questions** (14)

a) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form and find its rank. (5)

b) Using De Moivre's theorem prove that (5)

$$(i) \cos 5\theta = 5\cos \theta - 20\cos^3 \theta + 16\cos^5 \theta$$

$$(ii) \sin 5\theta = 5\sin \theta - 20\sin^3 \theta + 16\sin^5 \theta$$

c) If $\tan(\alpha + i\beta) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2\alpha = 1$. (4)

Q-8 **Attempt all questions** (14)

a) Investigate for what values of λ and μ the equations (5)

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

b) Find the continued product of all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. (5)

c) Find the inverse of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan reduction method. (4)

